WUGRAV 94-10

High-frequency oscillations of Newton's constant induced by Inflation

Paul J. Steinhardt

Department of Physics

University of Pennsylvania

Philadelphia, Pennsylvania 19104

Clifford M. Will

McDonnell Center for the Space Sciences, Department of Physics

Washington University

St. Louis, Missouri 63130

Abstract

We examine the possibility that an epoch of inflationary expansion induces high-frequency oscillations of Newton's constant, G. The effect occurs because inflation can shift the expectation value of a non-minimally coupled, Brans-Dicke-like field away from the minimum of its effective potential. At some time after inflation ends, the field begins to oscillate, resulting in periodic variations in G. We find conditions for which the oscillation energy would be sufficient to close the universe, consistent with all known constraints from cosmology and local tests of general relativity.

I. INTRODUCTION AND SUMMARY

Time-variation of Newton's constant, G, has been the subject of many investigations. Most studies, motivated by Dirac's large-number hypothesis, focus on monotonic variations. Brans-Dicke theory, for example, was devised as a field theoretic model that incorporates a monotonic variation in G. Its key element is a massless, scalar field, ϕ_{BD}, which couples non-minimally to the scalar curvature, $\tilde{R}$, through the interaction, $\phi_{BD}\tilde{R}$. The effective Newtonian constant is proportional to ϕ_{BD}^{-1}. The massless ϕ_{BD} increases uniformly in a matter dominated epoch, resulting in a monotonic decrease in G. The model is problematic from a quantum field theory point of view because there is no reason why the mass of ϕ_{BD} should remain zero after quantum corrections. The problem from an empirical point of view is that there are tight limits from cosmology and local tests of general relativity (for a review see [1])

In this paper, we consider the possibility of an *oscillating* Newtonian constant in which G varies with a frequency much greater than the expansion rate of the Universe, $\nu \gg 10^{-17}$ Hz [2]. A model with oscillations in G is quite simple to construct; *e.g.*, a massive Brans-Dicke model in which a scalar field Φ (related to ϕ_{BD}) is displaced from the minimum of its effective potential. The displacement in Φ could result naturally from an epoch of inflationary expansion. During inflation, the scalar curvature is non-zero, and so produces a substantial contribution to the effective potential for Φ through the ΦR interaction. The added contribution shifts the minimum of the effective potential for Φ. After reheating, the scalar curvature is zero, and Φ is free to oscillate about the true minimum of its effective potential. From the point of view of quantum field theory, the massive Brans-Dicke field does not suffer the naturalness problem of the massless case. One might anticipate that the mass of Φ would be similar to that of typical elementary particles, $m \geq 1$ GeV or so, a range corresponding to very high-frequency ($\nu^{-1} \sim m^{-1} << 1$ sec) oscillations.

Accetta and one of us [3] have previously discussed some effects that high-frequency oscillations of G could have on cosmological measurements. The purpose of the present

study is to examine a very plausible scenario in which inflation sets off the oscillations in G. For simplicity, we consider cosmological models for which the energy density contains three components: ordinary, baryonic matter, ρ_B; radiation ρ_R; and oscillatory energy, ρ_Φ. It is straightforward to extend our results to models which include a fourth, conventional dark matter component. We assume that some inflaton field completely independent of Φ is responsible for providing the vacuum energy density needed to drive inflation. We further assume that Φ is so weakly coupled that Φ-particles are not produced by the decay of the inflaton during reheating. During inflation, the Φ field is shifted from the minimum of its effective potential; after inflation, the only effect on oscillations is the Hubble red shift of the Φ-field kinetic energy. We find that the scenario passes all known tests for a wide range of model parameters. In particular, it is possible to find viable models in which the oscillatory energy in Φ provides all of the missing energy density needed to reach the critical density without introducing conventional dark matter.

The organization of the paper is as follows: in Section II, notation and the basic equations are introduced to describe massive Brans-Dicke cosmology with oscillating G. In Sections III and IV, the oscillations in G induced by an epoch of inflationary expansion are computed for scalar fields coupled to the scalar curvature through linear and quadratic interactions, respectively. The red shifting of the oscillation amplitude after inflation is analyzed to determine the oscillation energy today. It is shown that the oscillation energy is compatible with all cosmological constraints for a wide range of parameters and that it is possible for the oscillation energy to reach closure density. In Section V, the constraints from local tests of general relativity are examined, including measurements of light deflection and time delay, data from laser ranging to Earth-orbiting satellites and the Moon, and tests of Newtonian gravity using tower and laboratory experiments. Section VI presents concluding remarks.

II. BASIC EQUATIONS

A. Action and field equations

The gravitational theory underlying the oscillating-G cosmologies is a general scalar-tensor metric gravitational theory, in which the scalar field has both a mass and a non-linear $\lambda\phi^4$ self-interaction. The action has the general form

$$I = \int \sqrt{-\tilde{g}} d^4x [M^2 f(\phi)\tilde{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \tilde{V}(\phi)] + I_m(\psi_m, \tilde{g}_{\mu\nu}), \quad (2.1)$$

where $M^2 = (16\pi G)^{-1}$, $\tilde{g}_{\mu\nu}$ is the physical metric, I_m is the matter action, with matter fields ψ_m coupling only to $\tilde{g}_{\mu\nu}$ (hence a metric theory), and $\tilde{R}$ is the Ricci scalar constructed from $\tilde{g}_{\mu\nu}$. We use the standard notation and conventions of [1,4]. The potential $\tilde{V}$ is given by

$$\tilde{V}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 \,. \quad (2.2)$$

With $\tilde{V} = 0$, this would correspond to a generalized Brans-Dicke (BD) theory [1] with $\phi_{BD} \equiv f(\phi)$ and the BD coupling function

$$\omega = f/2(Mf')^2, \quad (2.3)$$

where "prime" denotes $d/d\phi$. The field equations are

$$\tilde{G}_{\mu\nu} = \frac{1}{2M^2 f}\tilde{T}_{\mu\nu} + \frac{1}{f}(f_{;\mu\nu} - \tilde{g}_{\mu\nu} \tilde{\Box} f) + \frac{1}{2M^2 f}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda} - \tilde{g}_{\mu\nu}\tilde{V}(\phi)), \quad (2.4a)$$

$$\tilde{\Box}\,\phi = \frac{fV' - 2f'V - \frac{1}{2}f'(1+6M^2 f'')\phi_{,\lambda}\phi^{,\lambda} + \frac{1}{2}f'\tilde{T}}{f + 3M^2(f')^2}, \quad (2.4b)$$

where $\tilde{T}_{\mu\nu}$ is the physical stress-energy tensor derived from the matter action $I_m(\psi_m, \tilde{g}_{\mu\nu})$, and $\tilde{\Box}$ is the d'Alembertian using the physical metric, and so on.

For cosmology, it turns out to be useful to recast the theory into a representation with an Einstein gravitational action by making a conformal transformation to the non-physical metric

$$g_{\mu\nu} \equiv f(\phi)\tilde{g}_{\mu\nu}. \quad (2.5)$$

This representation is sometimes called the Einstein conformal frame (see [5] for discussion). The action then takes the form

$$I = \int \sqrt{-g} d^4x [M^2 R - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi - V(\Phi)] + I_m(\psi_m, f^{-1} g_{\mu\nu}), \tag{2.6}$$

where $V(\Phi) = f^{-2}\tilde{V}(\phi)$ and Φ is defined by the differential equation

$$d\Phi/d\phi = f^{-1}(f + 3M^2 f'^2)^{1/2}. \tag{2.7}$$

Damour and colleagues [5–7] have discussed general classes of scalar-tensor theories in this framework; their scalar field φ is given by $\varphi = \Phi/2M$, $A(\varphi) = f^{-1/2}$, and $V = 0$. We will consider models in which the non-minimal coupling is linear, $f(\phi) = 1 + \alpha\phi/M$, and quadratic, $f(\phi) = 1 + \xi\phi^2/M^2$.

The field equations in this representation are

$$G_{\mu\nu} = \frac{1}{2M^2 f}\tilde{T}_{\mu\nu} + \frac{1}{2M^2}(\Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\Phi_{,\lambda}\Phi^{,\lambda} - g_{\mu\nu}V(\Phi)), \tag{2.8a}$$

$$\Box\Phi = \frac{d}{d\Phi}(V(\Phi) - \frac{1}{4}f^{-2}\tilde{T}) . \tag{2.8b}$$

where now $\Box$ and $G_{\mu\nu}$ are defined using the non-physical metric; however, $\tilde{T}_{\mu\nu}$ still refers to the physical stress-energy tensor defined using locally measured quantities and the physical metric.

B. Cosmology

Assuming a standard homogeneous and isotropic universe, we choose coordinates $(t, \mathbf{x})$ so that the "line element" constructed from $g_{\mu\nu}$ has the Robertson-Walker form

$$ds^2 = -dt^2 + a(t)^2 d\sigma^2, \tag{2.9}$$

where $a(t)$ is the scale factor, and $d\sigma^2$ is the spatial line element representing the hypersurfaces of homogeneity. Since we will assume a standard inflationary scenario for the early

universe, we will adopt a spatially flat metric, *i.e.* we will choose $k = 0$. The relation between t and a and the corresponding physical variables $\tilde{t}$ and $\tilde{a}$ in the Robertson-Walker version of the physical metric $\tilde{g}_{\mu\nu}$ is $d\tilde{t} = f^{-1/2}dt$, $\tilde{a} = f^{-1/2}a$. The physical density and pressure ρ and p have their usual meanings as local properties of the matter measured in local Lorentz frames of the physical metric $\tilde{g}_{\mu\nu}$. Defining $H \equiv \dot{a}/a$, where an overdot denotes derivative with respect to t, we obtain the cosmological field equations and equations of motion

$$H^2 = (6M^2)^{-1}\left(\rho f^{-2} + \frac{1}{2}\dot{\Phi}^2 + V(\Phi)\right), \tag{2.10a}$$

$$\ddot{\Phi} + 3H\dot{\Phi} = -dV(\Phi)/d\Phi - \frac{1}{4}(\rho - 3p)d(f^{-2})/d\Phi, \tag{2.10b}$$

$$\dot{\rho} = -3(\rho + p)(H - \frac{1}{2}\dot{f}/f). \tag{2.10c}$$

Note that the use of the non-physical metric g is the origin of the ρf^{-2} and the $-\frac{1}{2}\dot{f}/f$ terms in Eqs. (2.10a) and (2.10c), respectively. The physically measured Hubble parameter $\tilde{H} = \tilde{a}^{-1}d\tilde{a}/d\tilde{t}$ is related to the Hubble parameter used in the Einstein conformal representation by $\tilde{H} = f^{1/2}(H - \dot{f}/f)$.

If $\rho - 3p$ is Φ-independent, as we will assume in this paper, we can use Eq. (2.10b) to define an effective potential

$$\begin{aligned} V_{\text{eff}} &\equiv V(\Phi) + \frac{1}{4}(\rho - 3p)f^{-2} \\ &= f^{-2}(\tilde{V}(\phi) + \frac{1}{4}(\rho - 3p)), \end{aligned} \tag{2.11}$$

such that $\ddot{\Phi} + 3H\dot{\Phi} = -dV_{\text{eff}}/d\Phi$.

We assume that inflation is triggered by some inflaton field in the matter sector (not Φ!), whose vacuum energy density ρ_V dominates the total energy density and leads to an epoch during which $\rho_V = -p =$ constant. We relate the vacuum energy density to a temperature by the standard relation (in units where $\hbar = c = k_B = 1$)

$$\rho_V \equiv T_V^4 = (10^{14}\text{GeV})^4 t_V^4 \tag{2.12}$$

where $t_V \equiv T_V/10^{14}$GeV. Note that, with these definitions, $\rho_V/6M^4 = 2\times 10^{-18} t_V^4$.

In characterizing cosmological evolutions, it will be useful to define a number of key variables.

(a) *Oscillation amplitude:*

$$\eta \equiv f(\phi) - 1 = \begin{cases} \alpha\phi/M, & \text{[linear]} \\ \xi\phi^2/M^2, & \text{[quadratic]} \end{cases} \tag{2.13}$$

(b) *Coupling parameter:*

$$\beta \equiv 3M^2 f'^2 = \begin{cases} 3\alpha^2, & \text{[linear]} \\ 12\xi^2\phi^2/M^2, & \text{[quadratic]} \end{cases} \tag{2.14}$$

(c) *Anharmonicity parameter:*

$$\zeta \equiv \lambda\phi^2/2m^2, \tag{2.15}$$

(d) *Ratio of scalar-to-radiation energy density*

$$\chi \equiv \rho_\Phi / f^{-2}\rho_R\,, \tag{2.16}$$

where ρ_R is the energy density of radiation, and

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)\,. \tag{2.17}$$

The parameter ζ determines the relative importance of the $\lambda\phi^4$ anharmonic potential compared to the mass term $m^2\phi^2$. Note that β is related to the Brans-Dicke parameter ω by $\omega = \frac{3}{2}(1+\eta)/\beta$; as $\beta \to 0$ for finite η, $\omega \to \infty$, and the theory becomes effectively general relativistic.

For future use we also define the dimensionless coefficients

(e) Ratio of Planck mass to scalar mass:

$$y \equiv M/m = 1.7\times 10^{18}(1\text{GeV}/m)\,, \tag{2.18}$$

(f) Q

$$Q \equiv 12\lambda^{-1}(\rho_V/6M^4) = 2.4\times 10^{-17}\lambda^{-1}t_V^4\,. \tag{2.19}$$

We assume that the material content of the universe is strictly that of ordinary matter (no exotic dark matter), and we will treat the equation of state for matter as having three phases, inflationary, radiation dominated and matter dominated, with the ratio $p/\rho \equiv \nu =$ constant during each phase, with the respective values -1, 1/3, and 0.

In all our models we will assume that the present amplitude of G-oscillations is very small, *i.e.* $\eta_0 << 1$, where the sub- or superscript 0 denotes present values. We define density parameters at the present epoch in the Einstein conformal frame: for matter (baryons plus radiation): ${\Omega_m}^0 = \rho^0/6M^2H_0^2$; for radiation alone: ${\Omega_R}^0 = \rho_R^0/6M^2H_0^2$; for the scalar field: ${\Omega_\Phi}^0 = {\rho_\Phi}^0/6M^2H_0^2$. Because of the present smallness of η, and because of its rapid oscillations compared to the expansion timescale, the physically measured $\tilde{H}_0 \approx H_0$, so Ω_m^0, Ω_R^0, Ω_Φ^0 and χ_0 are interchangable with the physically measured quantities. We note that

$$\Omega_R^0 = 4.3 \times 10^{-5} h^{-2} (T_0/2.73)^4 \,, \tag{2.20}$$

where h is the present Hubble parameter in units of 100 km s^{-1} Mpc^{-1} ($0.4 < h < 1$), and T_0 is the temperature of the cosmic microwave background in degrees Kelvin.

Since Φ interacts only gravitationally , it is thermally decoupled from ordinary matter; we ignore the possibility of particle creation induced by the non-minimal coupling of ϕ to curvature. Consequently, we can relate the energy densities of baryons ρ_B and radiation ρ_R to the scale factor a and temperature T using the matter equation of motion in the physical representation, $d\rho/d\tilde{t} = -3(\rho + p)\tilde{H}$, combined with standard thermal equilibrium arguments. Thus, when baryons and radiation are decoupled, we have $\rho_B \propto \tilde{a}^{-3}$, $\rho_R \propto \tilde{a}^{-4}$, $T_R \propto \tilde{a}^{-1}$. Then, denoting the end of inflation, marked by the conversion of false vacuum energy density to radiation, by I, it is straightforward to show [8] that the non-physical scale factors at these epochs are related by

$$a_0/a_I = (T_I/T_0)(f_0/f_I)^{1/2} \approx 4 \times 10^{26} (f_0/f_I)^{1/2} (T_V/10^{14}\mathrm{GeV}) \,. \tag{2.21}$$

C. Evolution of oscillating scalar field

In the limit of small Φ, the basic oscillation frequency of Φ is $\sim m$. Roughly speaking, in an expanding cosmology, Φ will oscillate when the period of oscillation $\sim m^{-1}$ is small compared to the expansion timescale $\sim H^{-1}$, *i.e.* when $m >> H$. Because the equation for Φ is non-linear, the effective frequency of oscillation could differ substantially from m. As a consequence, we will find it necessary to give a refined criterion for the onset of oscillations, separately applicable for the linear and quadratic models which we are considering. Once oscillations commence, however, the subsequent evolution of the oscillation amplitude can be found using a generalization of the method used by Turner [9].

We first convert the field equation for Φ into an approximate "energy conservation" equation, by multiplying Eq. (2.10b) by $f^n\dot{\Phi}$ and extracting total time derivatives. The result is

$$\frac{d}{dt}[f^n(\frac{1}{2}\dot{\Phi}^2 + V(\Phi))] = -3Hf^n\dot{\Phi}^2 + 6M^2H^2 df^n/dt, \tag{2.22}$$

where $n = \frac{1}{2}(1-3\nu) = 2\,, 0\,, \frac{1}{2}$, for inflation, radiation domination and matter domination, respectively, and where we assume that n is constant during each phase dominated by the corresponding form of energy. We define

$$E = f^n(\frac{1}{2}\dot{\Phi}^2 + V(\Phi)), \tag{2.23a}$$

$$T = [\text{period of oscillation}] = \oint \frac{d\Phi}{\sqrt{2(f^{-n}E - V(\Phi))}}. \tag{2.23b}$$

Then for periods $<< H^{-1}$, we average the right-hand-side of Eq. (2.22) over a period, with the result

$$\langle 3H(f^n\dot{\Phi})^2\rangle \simeq (3H/T)\int_0^T f^n\dot{\Phi}^2 dt = (3H/T)\oint f^n\dot{\Phi}d\Phi, \tag{2.24a}$$

$$\langle 6M^2H^2df^n/dt\rangle \simeq 6M^2H^2T^{-1}\int_0^T (df^n/dt)dt \simeq 0. \tag{2.24b}$$

Defining $\gamma = (1/ET) \oint f^n \dot{\Phi} d\Phi$, we find

$$E^{-1} dE/dt = -3\gamma H = -3\gamma \dot{a}/a\,. \tag{2.25}$$

For $\gamma \simeq$ constant, this equation can be integrated to give $E \propto a^{-3\gamma}$. Defining the integrals

$$I_{\pm} = \oint (1 - f^n V(\Phi)/E)^{\pm 1/2} f^{n/2} d\Phi, \tag{2.26}$$

then

$$\gamma = 2I_+/I_-. \tag{2.27}$$

It is useful to write $I_{\pm}$ in terms of ϕ:

$$I_{\pm} = \oint (1 - f^{n-2}\tilde{V}(\phi)/E)^{\pm 1/2} (f + \beta)^{1/2} f^{n/2-1} d\phi. \tag{2.28}$$

Note that when Φ reaches its extremal values within each oscillation, $\dot{\Phi} = 0$, so $E = V(\Phi_m) = f(\phi_m)^{-2}\tilde{V}(\phi_m)$, where the subscript m denotes the extremal value. In the quadratic coupling case, there is a symmetry $\phi \to -\phi$, so the maximum and minimum values of ϕ are identical apart from a sign; in the linear coupling case, this is no longer necessarily true.

However, for ϕ oscillations generated by inflation, we will find that $\eta << 1$ already at the onset of oscillations, so that in evaluating γ, we can set $f \approx 1$. We can now treat $\phi_{max} = -\phi_{min} \equiv \phi_m$ in all cases. We define $\zeta_m \equiv \zeta(\phi_m)$, and $x \equiv \phi/\phi_m$. We also have, in the linear case, $\beta = 3\alpha^2$ and in the quadratic case, $\beta_m \equiv \beta(\phi_m)$. Then, independently of n,

$$I_{\pm}{}^{\text{linear}} = 4(1 + 3\alpha^2)^{1/2} \int_0^1 (1 - x^2)^{\pm 1/2} (1 + \frac{\zeta_m}{1 + \zeta_m} x^2)^{\pm 1/2} dx\,, \tag{2.29a}$$

$$I_{\pm}{}^{\text{quadratic}} = 4 \int_0^1 (1 - x^2)^{\pm 1/2} (1 + \frac{\zeta_m}{1 + \zeta_m} x^2)^{\pm 1/2} (1 + \beta_m x^2)^{1/2} dx\,. \tag{2.29b}$$

Table I shows the resulting values of γ for the appropriate ranges of ζ_m and β_m. For the linear case and the quadratic case with small coupling parameter β_m, E has the expected variation: as matter ($\propto a^{-3}$) in the harmonic regime, and as radiation ($\propto a^{-4}$) in the

anharmonic regime. However, for the quadratic case with large coupling parameter, E for anharmonic oscillations varies like matter ($\propto a^{-3}$), while for harmonic oscillations, it decreases more slowly than matter ($\propto a^{-2}$). The evolution of the amplitude ϕ_m can be obtained from the fact that $E \approx \frac{1}{2}m^2{\phi_m}^2(1+\zeta_m)(1+\eta_m)^{n-2}$. Table I shows the relevant values of δ, defined by $\phi_m \propto a^{-3\delta}$.

III. LINEARLY COUPLED OSCILLATING G

A. Oscillations generated by inflation

Inflation can naturally lead to a finite displacement of Φ from zero, leading to finite oscillations. We assume that a false vacuum epoch occurs during which $-p = \rho = \rho_V \simeq$ constant. Then $\ddot{\Phi} + 3H\dot{\Phi} = -dV_I(\Phi)/d\Phi$, where the effective inflationary scalar potential V_I is given by (Eq. (2.11))

$$V_I(\Phi) = \frac{\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \rho_V}{(1+\alpha\phi/M)^2}. \tag{3.1}$$

During the inflationary epoch, the effective damping caused by the $3H\dot{\Phi}$ term leaves the scalar field at the minimum of V_I, which occurs at $dV_I/d\Phi = 0 = dV_I/d\phi$.

Figure 1 shows schematically how the effective potential during inflation, $V_I(\Phi)$, differs from the effective potential today. The effect of the nonminimal couplings is to change the shape of the potential, shift the minimum away from $\Phi = 0$, and raise the value of the energy density at the minimum. The change in shape depends on the nature of the nonminimal coupling; *e.g.*, compare the upper and lower panels. For the linear case, we can use Eqs. (2.18) and (2.19), to obtain the equation for the minimum, expressed in terms of the amplitude of the oscillations in f, $\eta = \alpha\phi/M$,

$$\eta_I(\alpha^2/\lambda y^2 + \eta_I^2 + \frac{1}{2}\eta_I^3) = \alpha^4 Q. \tag{3.2}$$

The first term in Eq. (3.2) comes from $m^2\phi$ in $dV_I/d\phi$, the second comes from $\lambda\phi^3$, while the third comes from a cross term between $\lambda\phi^3$ and $\alpha\phi/M$ in the denominator of Eq. (3.1).

Note that, in this case, the anharmonicity parameter ζ (Eq. (2.15)), which is essentially the ratio between the second and first terms on the left-hand-side of Eq. (3.2), can be written $\zeta = \frac{1}{2}\lambda y^2\eta^2/\alpha^2$.

One class of solutions has the first term in parentheses in Eq. (3.2) dominant; this corresponds to $\zeta_I << 1$, *i.e.* to scalar fields in the harmonic regime during inflation. However, because ρ_Φ falls off as a^{-3} immediately, compared to the a^{-4} fall-off for radiation, the result can be shown to be unacceptable over-dominance of the scalar field energy density today, so we will not consider this case further (this case corresponds to the lower portion of Fig. 2). The alternative, anharmonic limit, $\zeta_I >> 1$ (second two terms dominant) yields

$$\eta_I^3(1+\frac{1}{2}\eta_I) = \alpha^4 Q, \tag{3.3}$$

with solutions

$$\eta_I \approx \begin{cases} \alpha^{4/3}Q^{1/3} << 1, & \text{for} \quad \alpha << 1.5\times 10^4\lambda^{1/4}t_V^{-1} \ , \\ 2^{1/4}\alpha Q^{1/4} >> 1, & \text{for} \quad \alpha >> 1.5\times 10^4\lambda^{1/4}t_V^{-1} \ . \end{cases} \tag{3.4}$$

The small α of the first case leads to a flatter potential (see Fig. 1), with a minimum at a relatively large value of ϕ, but a small η. The second case leads to a deeper potential, with a smaller, universal value at the minimum of $\phi/M \approx (24/\lambda)^{1/4}(\rho_V/6M^4)^{1/4}$, but a large value of η. For these solutions, generally $\eta_I/(1+\beta_I) = \eta_I/(1+3\alpha^2) << 1$ for λ and t_V of order unity (see Eq. (3.6) below).

B. Post-inflation evolution of scalar field

At the end of inflation when reheating occurs, $\dot{\Phi} \approx 0$, $\rho - 3p = 0$, and $\rho_\Phi \approx V(\Phi_I) = (\frac{1}{2}m^2\phi_I^2 + \frac{1}{4}\lambda\phi_I^4)/(1+\alpha\phi_I/M)^2$. For simplicity, we assume that the vacuum energy density is totally converted to radiation energy, so that $\rho_R = \rho_V$. Since we are considering the limit $\zeta_I >> 1$ we can approximate $V_I(\Phi) \approx f^{-2}(\frac{1}{4}\lambda\phi^4)$. Then using Eq. (3.3), we find that, right after inflation ends,

$$\chi_I \equiv (\rho_\Phi/f^{-2}\rho_R)_I \approx \eta_I^4/2\alpha^4 Q \approx \eta_I/(2+\eta_I), \tag{3.5a}$$

$$H^2 \approx (\rho_V/6M^2)(1+\eta_I)^{-1}(1+\frac{1}{2}\eta_I)^{-1}. \tag{3.5b}$$

However, when the false vacuum energy density is converted to radiation and $\rho - 3p \to 0$, the minimum of the potential V goes to $\Phi = 0$. Whether Φ begins to oscillate or rolls slowly down the potential V depends on whether the period of oscillation is short or long compared to the expansion time scale, where $(\text{period})^2 \approx \Phi(dV/d\Phi)^{-1} \approx \phi(d\Phi/d\phi)^2(dV(\Phi)/d\phi)^{-1}$. In this case, using $d\Phi/d\phi$ (Eq. (2.7)), we obtain $(\text{period})^2 \approx m^{-2}(1+\eta_I)(1+\beta+\eta_I)(1+2\zeta_I+\eta_I\zeta_I)^{-1}$. Since we have argued that $\eta_I/(1+\beta) << 1$, where $\beta = 3\alpha^2$, and $\zeta_I >> 1$, we then find the ratio

$$\mathcal{R} \equiv \frac{(\text{period})^2}{H^{-2}} \approx \left(\frac{1+3\alpha^2}{6\alpha^2}\right)\left(\frac{\eta_I}{2+\eta_I}\right). \tag{3.6}$$

For the solution with $\eta_I << 1$, $\mathcal{R} << 1$, so oscillations begin immediately, but with sufficiently small amplitude that we can approximate $f \approx 1$ right away. Then, initially, $\rho_\Phi \approx E \propto a^{-4}$, since the oscillations are anharmonic, and $\phi_m \propto a^{-1}$. Since $\zeta_m \propto \phi_m^2 \propto a^{-2}$, the transition to harmonic oscillations (H) at $\zeta_m \approx 1$ occurs at $a_H \approx \zeta_I^{1/2} a_I$. In the harmonic regime, $\rho_\Phi \propto a^{-3}$ and $\phi_m \propto a^{-3/2}$. It then follows that the present ratio of scalar to radiation energy densities is given by

$$\chi_0 = (\rho_\Phi/f^{-2}\rho_R)_0 \approx \chi_I \zeta_I^{-1/2}(a_0/a_I) \approx (2\lambda)^{-1/2}(\alpha/y)(a_0/a_I), \tag{3.7}$$

and the present amplitude of G-oscillations is $\eta_0 \approx \eta_I \zeta_I^{1/2}(a_I/a_0)^{3/2}$. Substituting Eq. (2.21) for a_0/a_I into Eq. (3.7), we obtain for the present scalar density parameter

$$\Omega_\Phi^0 h^2 = \Omega_R^0 h^2 \chi_0 \approx 10^{22}(2/\lambda)^{1/2}(\alpha/y)(T_V/10^{14}\text{GeV}), \tag{3.8}$$

where we have used Eq. (2.20) for the radiation density parameter and have assumed $T_0 = 2.73^\circ$ K. If G-oscillations are to account for the energy density needed to close the universe without other forms of dark matter, then we require $\Omega_\Phi^0 \approx 1$. For dimensionless coupling λ of order unity and $T_V \approx 10^{14}$GeV, the values of the scalar mass $m = M/y$ and the

coupling constant α that will meet this constraint are shown in Fig. 2. If the universe is flat, then $h < .65$ is needed to satisfy age constraints from globular cluster ages. In Fig. 2, the solid dark line corresponds to $\Omega_\Phi^0 h^2 = 0.25$, *e.g.*, $\Omega_\Phi^0 = 1$ and $h = 0.5$. The curve shows that Ω_Φ^0 near closure density is possible for a wide range of parameters, including the range where dimensionless parameters, such as λ and α of are of order unity. Notice in Fig. 2 that the present amplitudes of G-oscillation are extremely small, despite contributing a potentially critical energy density. Also plotted is the curve for $\Omega_\Phi^0 h^2 = 1$ (dashed curve, parallel to dark curve), which is, roughly, the upper bound on $\Omega_\Phi^0 h^2$ consistent with observations.

For the case $\eta_I >> 1$, corresponding to the right-hand strip of Fig. 2, the scalar energy density is comparable to the radiation energy density just following inflation (Eq. (3.5a)), and the ratio $\mathcal{R}$ (Eq. (3.5)) is of order unity. As a result, oscillations do not start immediately, instead, the scalar field rolls slowly down the potential, while the radiation energy density falls off rapidly. Eventually, η (and thus $\mathcal{R}$) becomes small enough that oscillations commence, first anharmonically, and then harmonically as before. But because of the intervening decrease in the radiation energy density, the resulting scalar energy density overdominates the universe today. This effect causes the curves of constant $\Omega_\Phi^0 h^2$ in Fig. 2 to turn up as they approach the line where $\eta_I \approx 1$ ($\alpha \approx 10^4 \lambda^{1/4} t_V^{-1}$).

IV. QUADRATICALLY COUPLED OSCILLATING G

A. Oscillations generated by inflation

In the quadratically coupled case, the effective inflationary scalar potential (Eq. (2.11)) is given by

$$V_I(\Phi) = \frac{\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \rho_V}{(1+\xi\phi^2/M^2)^2}. \tag{4.1}$$

Note that as $\phi \to \infty$, the potential tends to a constant, $\lambda M^4/4\xi^2$. Fig. 1 plots $V_I(\Phi)$ schematically. The minimum of the potential occurs at

$$\eta_I = 2\xi^2 Qr \begin{cases} << 1, & \xi << 10^8 \lambda^{1/2} t_V^{-2} r^{-1/2}\,, \\ >> 1, & \xi >> 10^8 \lambda^{1/2} t_V^{-2} r^{-1/2}\,, \end{cases} \tag{4.2}$$

where Q is given by Eq. (2.19), and

$$r \equiv \frac{1 - [24\xi y^2 \rho_V / 6M^4]^{-1}}{1 - \xi y^{-2} \lambda^{-1}} > 0. \tag{4.3}$$

If $r < 0$, the minimum occurs at $\eta_I = 0$, and no oscillations are generated by inflation. If $r > 0$, but both numerator and denominator in Eq. (4.3) are negative, then $\eta_I = 2\xi^2 Qr$ is a maximum, while $\eta_I = 0$ is a minimum, again resulting in no oscillations. These cases correspond to the region labelled "no oscillations" in Fig. 3. For the quadratically coupled model, the anharmonicity parameter is given by $\zeta = \frac{1}{2}\lambda y^2 \eta/\xi$. Note that for oscillations to occur, the numerator of Eq. (4.3) must be positive, thus $24\xi y^2 (H_I/M)^2 = \zeta_I/r > 1$, corresponding to the anharmonic regime. The larger ζ_I is, the closer r is to unity.

B. Post-inflation evolution of scalar field

In this case, we find at the end of inflation,

$$\chi_I \approx \eta_I^2 / 2\xi^2 Q \approx \eta_I, \tag{4.4a}$$

$$H^2 \approx (\rho_V / 6M^2)(1 + \eta_I r)(1 + \eta_I)^{-2}. \tag{4.4b}$$

The effective oscillation period is $(\text{period})^2 \approx m^{-2}(1+\eta_I)(1+\beta_I+\eta_I)(1+2\zeta_I-\eta_I)^{-1}$, Using Eq. (4.2) and the fact that $\zeta_I >> 1$, we find that the ratio $\mathcal{R}$ is

$$\mathcal{R} \approx \frac{1 + \beta_I + \eta_I}{24\xi r} \approx \frac{1 + (12\xi + 1)\eta_I}{24\xi}. \tag{4.5}$$

There are three possible post-inflation evolutions.

(a) Immediate Oscillations. Oscillations begin immediately when $\mathcal{R} << 1$, which corresponds to $1/24 < \xi < 2.9 \times 10^7 \lambda^{1/2}$. In this range, $\eta_I << 1$, so we approximate $f \approx 1$, and $\rho_\Phi << \rho_R$. However, although $\zeta_I >> 1$, $\beta_I = 12\xi\eta_I$ ranges from 10^{-18} to 10^{11}, with $\beta_I \approx 1$ for $\xi \approx 3 \times 10^5 \lambda^{1/3} t_V^{-4/3} r^{-1/3}$, so we must consider two separate cases (for the range

of parameters considered, $\beta_I/\zeta_I \approx 24\xi^2/y^2 << 1$). Recall that $\beta = \frac{3}{2}(1+\eta)/\omega$, so that large β corresponds roughly to small ω and to strong Brans-Dicke-like effects (we call this "strong non minimal coupling" or "strong coupling") and small β corresponds to large ω and to weak Brans-Dicke effects (we call this "weak coupling").

Case 1: $\beta_I >> 1$: Because of the strong non-minimal coupling, the energy density falls off like matter rather than like radiation despite the anharmonic potential (Table I), so that initially $E \approx \rho_\Phi \propto a^{-3}$, while β_m, ζ_m, $\eta_m \propto a^{-3/2}$; The transition to weak coupling (WC) ($\beta_m \approx 1$) occurs at $a_{WC} \approx \beta_I^{2/3} a_I$, whereupon standard anharmonic oscillations lead to $E \propto a^{-4}$ and β_m, ζ_m, $\eta_m \propto a^{-2}$. Harmonic oscillations commence at $a_H \approx \zeta_{WC}^{1/2} a_{WC} \approx \zeta_I^{1/2}\beta_I^{1/6} a_I$. During harmonic oscillations, $E \propto a^{-3}$ and β_m, ζ_m, $\eta_m \propto a^{-3}$. The result at the present epoch is

$$\chi_0 \approx \chi_I \zeta_I^{-1/2} \beta_I^{1/2} (a_0/a_I) \approx \eta_I \zeta_I^{-1/2} \beta_I^{1/2} (a_0/a_I), \tag{4.6a}$$

$$\eta_0 \approx \eta_I \zeta_I^{1/2} \beta_I^{1/2} (a_I/a_0)^3, \tag{4.6b}$$

and

$$\Omega_\Phi^0 h^2 \approx 4 \times 10^6 \lambda^{-3/2} (\xi^3/y)(T_V/10^{14}\mathrm{GeV})^5. \tag{4.7}$$

Note that, for larger values of ξ within the range quoted above, namely $7 \times 10^5 \lambda^{5/13} < \xi < 2.9 \times 10^7 \lambda^{1/2}$ the scalar energy density can actually surpass the radiation energy density during the strong-coupling regime. For the smaller values of ξ, the scalar density approaches but does not exceed the radiation during the strong-coupling regime; it is only when harmonic oscillations begin that the scalar energy surpasses the radiation. These two sub-cases are shown as curves (a) and (b) in Fig. 4.

Case 2: $\beta_I << 1$: In this case, weak coupling holds right after inflation. Standard anharmonic oscillations begin immediately, $E \propto a^{-4}$ and β_m, ζ_m, $\eta_m \propto a^{-2}$, with harmonic oscillations taking over when $a_H \approx \zeta_I^{1/2} a_I$ (curve (c) in Fig. 4). The evolution then proceeds as in the previous case, with the result

$$\chi_0 \approx \eta_I \zeta_I^{-1/2}(a_0/a_I), \tag{4.8a}$$

$$\eta_0 \approx \eta_I \zeta_I^{1/2}(a_I/a_0)^3, \tag{4.8b}$$

and

$$\Omega_\Phi^0 h^2 \approx 1.6 \times 10^{14} \lambda^{-1}(\xi^{3/2}/y)(T_V/10^{14}\mathrm{GeV})^3. \tag{4.9}$$

(b) Deferred oscillations. For $\xi << 1/24$, $\eta_I << 10^{-17}$, $\chi_I << 1$, but $\mathcal{R} >> 1$ and $\zeta_I >> 1$. This corresponds to small amplitudes for ϕ, though still in the anharmonic regime, but since $m/H << 1$ at this epoch, the scalar field first rolls slowly down the potential. Thus we can ignore $\ddot{\Phi}$ in Eq. (2.10b), and solve the approximate "slow-roll" evolution equation

$$\dot{\phi} = (d\phi/d\Phi)\dot{\Phi} \approx -(3H)^{-1}(d\phi/d\Phi)^2 dV(\Phi)/d\phi \approx -(m^2\phi^2/3H)(1+2\zeta). \tag{4.10}$$

Radiation strongly dominates the energy density following inflation, so $H \sim 1/2t$. Noting that $\zeta \propto \phi^2$, we rewrite Eq. (4.10) in the form $\dot{\zeta} \approx -\frac{4}{3}m^2 t\zeta(1+2\zeta)$. For $mt \sim m/H << 1$ we find the approximate solution $\zeta \approx \zeta_I/(1+\frac{4}{3}\zeta_I m^2 t^2)$. Noting that $m^2\zeta_I \approx 12\xi(\rho_V/6M^2)$ and defining $\tau \equiv t(\rho_V/6M^2)^{1/2}$, we obtain

$$\mathcal{R} \approx (2t)^{-1}(2m^2\zeta)^{-1} \approx [1+16\xi\tau^2]/96\xi\tau^2. \tag{4.11}$$

Thus oscillations begin when $\tau \approx (80\xi)^{-1/2}$. At this point, $\rho_R \approx 2\rho_I\tau^{-2} \approx 20\xi\rho_I$, $\rho_\Phi \approx \lambda\phi^4 \approx (25/36)\rho_\Phi^I$, and $a \approx a_I/(20\xi)^{1/4}$. Following the oscillations from this point onward, we find

$$\Omega_\Phi^0 h^2 \approx 10^{13}\lambda^{-1}(\xi^{3/4}/y)(T_V/10^{14}\mathrm{GeV})^3. \tag{4.12}$$

This corresponds to curve (d) in Fig. 4.

(c) Lingering inflation. For $\xi >> 3 \times 10^8 \lambda^{1/2}$, $\eta_I >> 1$, we have $\rho_\Phi \approx (\lambda/4\xi^2)M^4$, $\chi_I \approx \eta_I >> 1$, and $\mathcal{R} \approx \eta_I/2 >> 1$. In this case, the scalar-field energy density dominates, and the scalar field rolls slowly down the flat wing of the post-inflation potential (Figure 1). In this case, the slow-roll evolution equation gives

$$\dot{\phi} \approx -(3H)^{-1}(d\phi/d\Phi)^2 dV(\Phi)/d\phi \approx -\frac{1}{3}(\lambda/6)^{1/2}\xi^{-2}M^3\phi^{-1}, \tag{4.13}$$

where we have used $H \approx (\lambda/24)^{1/2}\xi^{-1}M$, and $\beta,\ \eta,\ \zeta >> 1$. The solution is $\eta_I - \eta \approx \frac{1}{3}(\lambda/6)^{1/2}\xi^{-1}M(t-t_I)$. Oscillations in Φ commence when $\eta \approx 1 << \eta_I$, *i.e.* when $H(t-t_I) \approx \frac{3}{2}\eta_I$. During this period, $H \approx$ constant, $a \propto e^{Ht}$, and the radiation density deflates by $e^{-4H(t-t_I)} \approx e^{-6\eta_I}$. This is a kind of "lingering inflation", driven by the approximately constant scalar energy density on the flat portion of the potential at large ϕ (Figure 1). Eventually oscillations begin and the evolution proceeds as in the previous case, except that the lingering inflation leads to an effective initial χ_I larger than before by the factor $e^{6\eta_I} >> 1$. The result is a much larger value for the scalar-field energy density:

$$\Omega_\Phi^0 h^2 \approx 4\times 10^6 \lambda^{-3/2}(\xi^3/y)e^{(4\times 10^{-16}\lambda^{-1}\xi^2 t_V^4)}(T_V/10^{14}\mathrm{GeV})^5. \tag{4.14}$$

Because $\xi > 3\times 10^8 \lambda^{1/2}$ in this case, we see that the scalar field overdominates the present energy density by many orders of magnitude. This range of values of ξ is therefore unviable.

As discussed for the linear case, $\Omega_\Phi^0 h^2 \approx 0.25$ corresponds to the case where the oscillation energy in Φ is near the closure density, assuming no additional dark matter. Imposing $\Omega_\Phi^0 h^2 \approx 0.25$ and assuming λ and $T_V/10^{14}$GeV of order unity, we obtain from Eqs. (4.7), (4.9), (4.12), and (4.14) the constraint on the scalar mass $m = M/y$ and the coupling constant ξ shown in Fig. 3. As with the linear case, we find that Ω_Φ^0 near closure density is possible for a wide range of parameters, including the range where dimensionless parameters, such as λ and ξ of are of order unity. Also shown is the corresponding curve for $\Omega_\Phi^0 h^2 = 1$, roughly the largest value compatible with observations.. Not surprisingly, given the quadratic nature of the non-minimal coupling, the amplitude of G-oscillations at the current epoch is extraordinarily small, despite their ability to generate a closure energy density.

V. CONSTRAINTS FROM LOCAL EXPERIMENTS

We turn now to a discussion of bounds on these models imposed by local laboratory and solar-system experiments. Because of the presence of the potential $V(\Phi)$, there is a

length scale ℓ over which modifications to general relativity may be important. For scales much greater than ℓ, the theory is equivalent to general relativity. Numerous tests of general relativity and of the inverse-square law of gravity using a variety of techniques have been performed which cover a range of lengths from solar-system scales down to centimeters. In this section we present a general method for using the results of such experiments to bound the parameters of oscillating G theories. We will find that the most viable models from a cosmological viewpoint are comfortably consistent with all local experimental limits.

Comparison with experiment is most easily done in the physical representation. We expand the physical metric and the scalar field about their asymptotic values, $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and $\phi = \phi_0(t) + \varphi$, and write $f(\phi) = f_0 + f_0'\varphi + \frac{1}{2}f_0''\varphi^2$, and substitute into Eqs. (2.4), keeping terms to first order in $h_{\mu\nu}$ and φ. In eliminating the zeroth order terms in the field equation for ϕ, corresponding to the cosmological solution for ϕ_0, we note that the coordinate t used here is related to the time $t_{\rm RW}$ of the physical Robertson-Walker metric by $dt_{\rm RW} = (1 - \frac{1}{2}h_{00})dt$. The result is

$$-\ddot{\varphi} + (\nabla^2 - \tilde{m}^2)\varphi = -\frac{1}{2}\frac{f_0'}{f_0 + 3M^2 f_0'^2}(\rho - 3p) + \frac{1}{2}\frac{f_0'(1 + 6M^2 f_0'')}{f_0 + 3M^2 f_0'^2}\dot{\phi}_0\dot{\varphi} + \frac{1}{2}\dot{h}_{ii}\dot{\phi}_0, \tag{5.1}$$

where

$$\begin{aligned}\tilde{m}^2 = \frac{m^2}{f_0 + 3M^2 f_0'^2}\Big\{ & f_0 - f_0'\phi_0 - f_0''\phi_0^2 - \frac{f_0'\phi_0(f_0 - f_0'\phi_0)(1 + 6M^2 f_0'')}{f_0 + 3M^2 f_0'^2} \\ & -\frac{1}{2}\frac{\dot{\phi}^2}{m^2}\frac{1 + 6M^2 f_0''}{f_0 + 3M^2 f_0'^2}(f_0'^2 - f_0 f_0'' + 3M^2 f_0'^2 f_0'')\Big\}.\end{aligned} \tag{5.2}$$

We are interested in approximately static solutions corresponding to a gravitating mass such as the Sun or Earth, approximately at rest ($v \approx 0\,, p << \rho$). Thus we can drop the terms involving time derivatives. Using Eq. (2.3), and averaging the oscillations of ϕ_0 over several periods, we find

$$(\nabla^2 - \tilde{m}^2)\varphi = -\left(\frac{f_0'}{f_0}\right)\frac{\omega}{(3 + 2\omega)}\rho, \tag{5.3}$$

whose solution is

$$\varphi = 4M^2 f_0' \frac{\mu}{r} e^{-\tilde{m}r}, \tag{5.4}$$

where $\mu = f_0^{-1} G M_{\text{source}}$ is the effective gravitational mass of the gravitating body.

Using the approximations $f_{;00} \approx 0$, $\Box f \approx f_0' \nabla^2 \varphi$, $T_{00} \approx \rho$, $T_{ij} \approx 0$, and working in a gauge in which $h^{\mu}_{i\ ,\mu} - \frac{1}{2} h^{\mu}_{\mu,i} = (f_0'/f_0)\varphi_{,i}$, we obtain the field equations

$$\nabla^2 (h_{00} - (f_0'/f_0)\varphi) = -\rho/2M^2 f_0 - (\rho_\phi + 3p_\phi)/2M^2 f_0, \tag{5.5a}$$

$$\nabla^2 (h_{ij} + (f_0'/f_0)\delta_{ij}\varphi) = -\delta_{ij}\rho/2M^2 f_0 - \delta_{ij}(\rho_\phi - p_\phi)/2M^2 f_0, \tag{5.5b}$$

where $\rho_\phi = \frac{1}{2}\dot{\phi}_0^2 + \tilde{V}(\phi_0)$ and $p_\phi = \frac{1}{2}\dot{\phi}_0^2 - \tilde{V}(\phi_0)$. Dropping the ρ_ϕ and p_ϕ terms for the moment, we obtain the solutions

$$h_{00} = \frac{2\mu}{r}\left(1 + \frac{1}{3+2\omega} e^{-\tilde{m}r}\right), \tag{5.6a}$$

$$h_{ij} = \frac{2\mu}{r}\delta_{ij}\left(1 - \frac{1}{3+2\omega} e^{-\tilde{m}r}\right). \tag{5.6b}$$

The effective mass $\tilde{m}$ leads to a range $\tilde{m}^{-1}$ for modifications of Einstein gravity. The "Newtonian" potential h_{00} can be written in the more familiar form $h_{00} = 2G_{eff}\mu/r$, where G_{eff} has the effective form

$$G_{eff} = \left(1 + \frac{1}{3+2\omega} e^{-\tilde{m}r}\right). \tag{5.7}$$

Because of the Yukawa-type correction to the potential, for $r << \tilde{m}^{-1}$, $G_{eff} \approx (4+2\omega)/(3+2\omega)$, while for $r >> \tilde{m}^{-1}$, $G_{eff} \approx 1$. Thus, for $r >> \tilde{m}^{-1}$, the theory is equivalent to general relativity. Experimental tests of classical general relativity can in principle be used to place bounds on $\tilde{m}$ and on the coupling parameters α and ξ of the two oscillating G models.

A. Light deflection and time delay

To lowest order in μ/r, the physical metric obtained from Eqs. (5.6) can be written in the form

$$g_{00} \approx -(1 - \frac{2\mu}{r})(1 - \frac{2\mu}{3+2\omega}\frac{e^{-\tilde{m}r}}{r}), \tag{5.8a}$$

$$g_{ij} \approx \delta_{ij}(1 + \frac{2\mu}{r})(1 - \frac{2\mu}{3+2\omega}\frac{e^{-\tilde{m}r}}{r}). \tag{5.8b}$$

Notice that, in the line element, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, the exponential terms can be extracted as an overall conformal factor. Hence, there is no direct effect on null geodesics, and the deflection and time delay of a signal is proportional to μ, with no explicit dependence on $\tilde{m}$. However, μ represents the mass as measured at spatial infinity using Kepler's law. In solar system applications, such as the deflection of light, the solar mass actually used is measured by using Kepler's law applied to the Earth's orbit. If the range $\tilde{m}^{-1}$ is small compared to the Earth's orbit, *i.e.* $\tilde{m}^{-1} << 10^8$km, corresponding to $\tilde{m} >> 10^{-27}$ GeV, or an oscillation frequency $\nu >> 10^{-2}$ Hz, the inferred solar mass $M_\odot$ is the same as μ in Eq. (5.8), and consequently, the predicted time delay and light deflection in terms of the measured solar mass are indistinguishable from Einstein relativity. This corresponds to $y << 10^{45}$, which easily includes the masses of interest for cosmology. Conversely, if $\tilde{m}^{-1} >> 10^8$ km, the inferred mass is $M_\odot = \mu(4 + 2\omega)/(3 + 2\omega)$, and the prediction for time-delay and light deflection approaches the Brans-Dicke form, proportional to $(3 + 2\omega)/(4 + 2\omega)$.

B. Orbits

Using Earth-LAGEOS-Lunar data [10] which compares $GM_\oplus$ inferred from Earth-surface gravity measurements to the values inferred from the study of the orbits of the LAGEOS satellites and the Moon, tight bounds can be obtained on the variation in G in the 10^4 – 10^8 m range. From a rough fit to the appropriate portion of Fig. 1 in [11], we find that the coefficient in the exponential correction in Eq. (5.7) is constrained to be

$$(3 + 2\omega)^{-1} < 0.03(\nu/\text{MHz})^{1.1}, \tag{5.9}$$

for frequencies in the 10 kHz to 1 Hz range, corresponding to values of y between 10^{38} and 10^{42}. For the higher frequencies of interest to our cosmological models, the bound is easily satisfied.

C. Tower and laboratory measurements

Tower measurements of the variation in G place the limit $(3+2\omega)^{-1} < 2\times 10^{-4}$ at a range of 100 meters *i.e.* $\nu = 3$ MHz, or $y = 10^{36}$. Stronger limits are obtained from laboratory tests. From a null test of Gauss's law at 1.5 meters using superconducting gravity gradiometers [12] ($\nu = 200$ MHz, $y = 10^{34}$), we find the bound

$$(3+2\omega)^{-1} < 3\times 10^{-5}\ell^2 e^{3/\ell}, \tag{5.10}$$

where ℓ is the range in meters. From a test of the inverse square law at 2 centimeters using a torsion balance [13] ($\nu = 15$ GHz, $y = 10^{32}$), we obtain

$$(3+2\omega)^{-1} < 3.4\times 10^{-2}\ell^2 e^{0.04/\ell}. \tag{5.11}$$

The bounds relax exponentially with increasing mass and frequency (decreasing range), and quadratically with decreasing mass and frequency (increasing range). Notice that the conversion from range ℓ to the variable y is $\ell/1\text{meter} = 1.14\times 10^{-34} y(m/\tilde{m})$. Since the range of values of y of interest to cosmology is $y < 10^{30}$, only the bound from the shortest-range experiment, Eq. (5.11) will be meaningful.

D. Application to specific models

Linear Coupling. With small amplitude oscillations at the present epoch ($\eta_0 << 1$), we find from Eqs. (2.3) and (5.2)

$$\omega = 1/2\alpha^2\,, \;\; \tilde{m}^2 = m^2/(1+3\alpha^2). \tag{5.12}$$

For $y < 10^{30}$, Eq. (5.11) yields the bound $\alpha y < 4\times 10^{31}$. This bound appears in the upper right corner of Fig. 2.

Quadratic Coupling. In this case, to first order in η_0 and β_0, we have

$$\omega = 3/2\beta_0\,, \;\; \tilde{m}^2 = m^2(1-5\eta_0-2\beta_0). \tag{5.13}$$

But because η_0 and β_0 are so small (see Fig. 3), ω is sufficiently large to be compatible with any experimental bounds; in this case, despite the massive scalar field, general relativity is a strong attractor at the present epoch.

E. Effective cosmological constant

We now consider the terms in Eqs. (5.5) that involve the energy density and pressure of the oscillating scalar field. For harmonic oscillations, averaged over a period, $\langle\rho_\phi\rangle \approx$ constant, varying only on a Hubble timescale, while $\langle p_\phi\rangle \approx 0$. The resulting spatially uniform cosmological scalar energy density acts like an effective cosmological constant, resulting in contributions to the local physical metric of the form

$$g_{00} \approx -1 - \Omega_\phi^0 H_0^2 r^2\,, \tag{5.14a}$$

$$g_{ij} \approx \delta_{ij}(1 - \Omega_\phi^0 H_0^2 r^2)\,. \tag{5.14b}$$

These terms will modify the gravitational dynamics of local systems, ranging from the solar system to clusters of galaxies. In the case of the usual cosmological constant Λ, observational bounds from such systems are generally not sufficient to constraint the energy density represented by Λ to values below the critical energy density [8]. Thus those observations are consistent with our desire to have $\Omega_\Phi^0 \approx 1$.

VI. CONCLUSIONS

The scenario we have investigated in this paper links together several plausible components. First, we have assumed some modification of standard, Einstein gravity in the form of a non-minimally coupled scalar field. Modifications are required in almost any attempt to obtain a unified quantum theory of particle physics and general relativity, including supergravity and superstring theories. The specific non-minimal coupling considered in this paper is similar to the coupling of the dilaton. Second, we have assumed that the non-minimally

coupled field is massive and has typical self-interactions of quantum scalar fields. Third, we have assumed inflation, which is well-motivated for many independent reasons.

We have shown that, taken together, these components lead naturally to a scenario in which G oscillates about its mean value. Inflation shifts the scalar field away from its minimum. At some time after inflation, the field begins to oscillate. If the mass of the scalar field is typical of elementary particles, $m \geq 1$ GeV or so, the oscillations are high frequency compared to the expansion rate of the Universe. The amplitude is exponentially small, though, since the Hubble expansion steadily redshifts the kinetic energy density between inflation and the present epoch. For a wide range of model parameters, the combination of the high frequency and small amplitude that typically arises passes all known tests from cosmology and general relativity.

Our analysis applies to models in which the Φ field evolves purely according to the classical equations of motion (2.10b) and the oscillation energy dissipates through Hubble red shift. Since the Φ-field is nonminimally coupled, it is possible in many theories for the Φ-field energy to dissipate also through decay into other particles via gravitational interactions. Decay commences once the age of the Universe exceeds the lifetime of the Φ-field. The classical calculations leading to Figs. 2 and 3 significantly overestimate the oscillation amplitude and energy today. The possibilities depend upon the details of the model and lie beyond the scope of this paper.

The provocative possibility raised by the analysis is that the oscillation energy in G could account for a large fraction of the energy density needed to close the Universe. Here, even the most extreme case, where oscillation energy is the only nonluminous energy density, has been shown to be viable for a wide range of parameters, including the range for which dimensionless parameters are of order unity. A future challenge is to determine if oscillation energy by itself or combined with conventional dark matter can account for large-scale structure formation. Perhaps these investigations will motivated new experimental approaches for measuring high frequency oscillations of Newton's constant, or other local effects of such theories.

ACKNOWLEDGMENTS

This research was supported in part by the Department of Energy Grant DOE-EY-76-C-02-3071 at the University of Pennsylvania, and by the National Science Foundation, Grant Nos. 89-22140 and 92-22902 at Washington University.

REFERENCES

[1] C. M. Will, *Theory and Experiment in Gravitational Physics*, Revised Edition (Cambridge University Press, Cambridge, 1993).

[2] This is to be contrasted with models with frequency much smaller than the expansion rate, invoked in efforts to explain periodicities in pencil-beam galaxy redshifts surveys. See M. Morikawa, Astrophys. J. Lett. **362**, L37 (1990); C. T. Hill, P. J. Steinhardt, and M. S. Turner, Phys. Lett. B **252**, 343 (1990); R. G. Crittenden and P. J. Steinhardt, Astrophys. J. **395**, 360 (1992).

[3] F. S. Accetta and P. J. Steinhardt, Phys. Rev. Lett. **67**, 298 (1991).

[4] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[5] T. Damour and G. Esposito-Farèse, Class. Quantum Grav. **9**, 2093 (1992).

[6] T. Damour and G. Esposito-Farèse, Phys. Rev. Lett. **70**, 2220 (1993).

[7] T. Damour and K. Nordtvedt, Phys. Rev. Lett. **70**, 2217 (1993); Phys. Rev. D **48**, 3436 (1993).

[8] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley Publishing Co., Redwood City, 1990).

[9] M. S. Turner, Phys. Rev. D **28**, 1243 (1983).

[10] R. H. Rapp, Geophys. Res. Lett. **14**, 730 (1987).

[11] E. Fischbach and C. Talmadge, Nature **356**, 207 (1992).

[12] M. V. Moody and H. J. Paik, Phys. Rev. Lett. **70**, 1195 (1993).

[13] J. K. Hoskins, R. D. Newman, R. Spero, and J. Schultz, Phys. Rev. D **32**, 3084 (1985).

FIGURES

FIG. 1. Schematic plot of effective inflationary scalar potential during and after inflation

FIG. 2. Allowed values of $y = M/m$ and α in the linearly coupled model. Hatched line corresponds to bounds from laboratory inverse square-law tests. The solid dark line corresponds to $\Omega_\Phi^0 h^2 = 0.25$, the case of a critical density of scalar field energy today and $h = 0.5$. Also shown is $\Omega_\Phi^0 h^2 > 1$ (dashed line and shading), roughly the upper limit consistent with present observations. Dotted lines are lines of constant amplitude of present-day G-oscillations (η_0). Also shown (dashed lines with shading) are the excluded regions in which $\eta_I \gg 1$ or $\zeta_I \ll 1$.

FIG. 3. Allowed values of $y = M/m$ and ξ in the quadratically coupled model. Here, the oscillation amplitude (η_0) is so tiny that laboratory experiments provide no useful bounds. The excluded regions are where the field is not displaced from the minimum of the effective potential during inflation (bottom shaded region) or where there is lingering inflation due to the Φ field such that oscillation energy ultimately overdominates the universe.

FIG. 4. Schematic evolution of scalar energy density, ρf^2, in quadratically coupled model for various ranges of coupling parameter ξ and mass m, all leading to the same scalar energy density today. Superimposed are solid curves showing the evolution of the baryon density, ρ_B, and the radiation energy density, ρ_R. The scalar energy density corresponds to the various dashed curves: Curves (a) and (b): Oscillations begin immediately after inflation, but in the strong nonminimally coupled regime, where $\rho f^2 \propto a^{-3}$. Depending on value of ξ scalar energy density may grow to exceed that of the radiation density, *e.g.*, curve (a), before entering the weakly-coupled regime where ρf^2 begins to evolve as a^{-4}. Curve (c): Immediate oscillations in weakly-coupled regime fall off as radiation from the start. Curve (d): Deferred oscillations cause scalar energy density to remain roughly constant initially, then to fall off like radiation. Eventually harmonic oscillations take over in all cases, leading to matter-like falloff. In all cases, the late, weakly-coupled behavior switches to harmonic oscillations with $\rho f^2 \propto a^{-3}$.

TABLES

TABLE I. Values of γ and δ for scalar-field oscillations varying according to $E \propto a^{-3\gamma}$ and $\phi_m \propto a^{-3\delta}$

	Linear	Quadratic	
		$\beta_m << 1$	$\beta_m >> 1$
	Values of γ		
Harmonic $\zeta_m << 1$	1	1	2/3
Anharmonic $\zeta_m >> 1$	4/3	4/3	1
	Values of δ		
Harmonic $\zeta_m << 1$	1/2	1/2	1/3
Anharmonic $\zeta_m >> 1$	1/3	1/3	1/4